Erratum: Precession of a planet with a satellite

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Abstract. We correct a mistake in the proof of the proposition 1, given in section 4 of the paper *Icarus*, 185 (2006) 312-330 (Paper I). The proof is slightly modified but the results remain identical.

In section 4 (Global solution), $\mathcal{W}$ is the matrix $(\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2)$ and V the Gram matrix of the basis $(\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2)$

$$V = \begin{pmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{pmatrix} \ , \tag{1}$$

We have incorrectly stated that the averaged differential system (51) of Paper I

$$\begin{aligned} \dot{\mathbf{w}} &= -\frac{\mathfrak{a}x}{\gamma}\mathbf{w}_1 \wedge \mathbf{w} \ -\frac{\mathfrak{b}y}{\gamma}\mathbf{w}_2 \wedge \mathbf{w} \ , \\ \dot{\mathbf{w}}_1 &= -\frac{\mathfrak{c}z}{\beta}\mathbf{w}_2 \wedge \mathbf{w}_1 \ -\frac{\mathfrak{a}x}{\beta}\mathbf{w} \wedge \mathbf{w}_1 \ , \\ \dot{\mathbf{w}}_2 &= -\frac{\mathfrak{b}y}{\alpha}\mathbf{w} \wedge \mathbf{w}_2 \ -\frac{\mathfrak{c}z}{\alpha}\mathbf{w}_1 \wedge \mathbf{w}_2 \ , \end{aligned} \tag{2}$$

can be written as

$$\dot{\mathcal{W}} = \mathcal{W}\mathcal{B} \qquad \text{where} \qquad \mathcal{B} = vV^{-1}\mathcal{A} \tag{3}$$

is a matrix depending only on (x, y, z). In fact, the correct expression is (see Appendix B of Paper I)

$$\dot{\mathcal{W}} = vV^{-1}\mathcal{W}\mathcal{A} \tag{4}$$

which cannot be reduced to the previous one. As vV^{-1} and $\mathcal{A}$ depend only on (x, y, z) that are periodic functions of period T, it is still true that if $\mathcal{W}(t)$ is a solution of (4), then $\mathcal{W}(t+T)$ is also a solution, but the remaining part of the proof has to be modified, as Eq. (96) of Paper I is no longer correct. Let us still denote

$$\mathcal{R}_T(t) = \mathcal{W}(t+T)\mathcal{W}(t)^{-1} \ . \tag{5}$$

We need to prove that $\mathcal{R}_T(t)$ is constant with t. As the Gram matrix V of the vectors $(\mathbf{w}(t), \mathbf{w}_1(t), \mathbf{w}_2(t))$ is T-periodic, the norm is conserved by the linear transformation $\mathcal{R}_T(t)$ that send $\mathcal{W}(t)$ into $\mathcal{W}(t+T)$, and $\mathcal{R}_T(t)$ is thus an isometry of $\mathbb{R}^3$. Moreover, this isometry is positive, as the volume v is conserved over a full period T (see section 3.4 of Paper I). The invariance of the total angular momentum $\boldsymbol{W}_0$ (Eq. 52 of Paper I) then implies that $\mathcal{R}_T(t)$ is a rotation matrix of axis $\boldsymbol{W}_0$.

As $\mathcal{R}_T(t)$ is a rotation in $\mathbb{R}^3$, we have for all $\mathbf{w}_i, \mathbf{w}_j \in \{\mathbf{w}, \mathbf{w}_1, \mathbf{w}_2\}$,

$$\begin{aligned} \mathbf{w}_i(t+T) \wedge \mathbf{w}_j(t+T) &= (\mathcal{R}_T(t)\mathbf{w}_i(t)) \wedge (\mathcal{R}_T(t)\mathbf{w}_j(t)) \\ &= \mathcal{R}_T(t)(\mathbf{w}_i(t) \wedge \mathbf{w}_j(t)) \ . \end{aligned} \tag{6}$$

From Eq. (2), we can thus derive

$$\dot{\mathcal{W}}(t+T) = \mathcal{R}_T(t)\dot{\mathcal{W}}(t) \ . \tag{7}$$

On the other hand, as $\mathcal{W}(t+T) = \mathcal{R}_T(t)\mathcal{W}(t)$ (Eq. 5), we deduce that for all t,

$$\dot{\mathcal{R}}_T(t)\mathcal{W}(t) = 0 \ . \tag{8}$$

$\mathcal{R}_T(t)$ is thus a constant matrix $\mathcal{R}_T$. The last part of the proof remains identical. Let us denote $\mathcal{R}(t)$ the rotation of axis $\boldsymbol{W}_0$ and angle $t\theta_T/T$ (i.e. $\mathcal{R}(T) = \mathcal{R}_T$). We have

Proposition 1. The complete solution $\mathcal{W}(t)$ can be expressed in the form

$$\mathcal{W}(t) = \mathcal{R}(t)\tilde{\mathcal{W}}(t) \ , \tag{9}$$

where $\tilde{\mathcal{W}}(t)$ is periodic with period T, and $\mathcal{R}(t)$ a uniform rotation of axis $\boldsymbol{W}_0$ and angle $t\theta_T/T$. The motion has two periods: the (usually) short period T and the precession period

$$T' = \frac{2\pi}{\theta_T}T \ . \tag{10}$$

Miscellaneous misprints

We take the opportunity to also report here some misprints that remain in the published version of Paper I :

Eq. (3) of Paper I should read

$$\begin{pmatrix}\tilde{\mathbf{r}}_0\\ \tilde{\mathbf{r}}_1\\ \tilde{\mathbf{r}}_2\end{pmatrix} = \begin{pmatrix}1 & 1 & 1\\ 0 & 1 & 1\\ 0 & -\delta & 1-\delta\end{pmatrix} \begin{pmatrix}\tilde{\mathbf{u}}_0\\ \tilde{\mathbf{u}}_1\\ \tilde{\mathbf{u}}_2\end{pmatrix} \ .$$

Appendix A. Averaged quantities
The averaged value of $\mathbf{r}/r^5$ is

$$\langle\frac{\mathbf{r}}{r^5}\rangle = +\frac{e}{a^4(1-e^2)^{5/2}}\boldsymbol{i} \ .$$

Reference :
Boué, G., Laskar, J.: 2006, Precession of a planet with a satellite, *Icarus*, **185**, 312–330